# WAVES – A LESSON PLAN

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Level: Generally, 11th grade high school through college
Purpose: To give an introductory to in depth view of wave motion – its theory and applications

## INTRODUCTION – WHAT ARE WAVES AND HOW ARE THEY IMPORTANT?

When we hear about **waves**, we tend to think of water, sound and light. But the concept is much more far reaching and explains a huge number of phenomena from how music is produced to the esoterica of quantum mechanics.

This lesson plan is designed to help teachers to introduce students to the main *concepts* involved in waves and lead from these ideas to *applications*.

I give tips every so often to help you decide how to select and present topics.

This lesson plan is multilevel, so parts of it can be skipped, as is appropriate to student preparation and your emphasis.

## PERIODICITY AND PERIOD – CONCEPTS ESSENTIAL TO UNDERSTANDING WAVES (AND MANY OTHER PHENOMENA)

The needed background is for students to have a basic understanding of *functions* and how they are graphed.

**Periodicity** arises from the concept of a **periodic function**.

**TIP:** You might want to start by asking your students for *their* definitions.

Assume a function graphed as $y = f(x)$ has this property: In words, it repeats itself every time *x* changes by a certain amount. The *smallest* change in *x* which causes a repeat is called **the period** of the function. The function is then called a **periodic function**.

Here is a more formal approach. Denoting the period by *p*, this means that, for *any x* in the domain of the function

$$f(x) = f(x+p) = f(x+2p) = \ldots f(x+np)\ldots$$

where all the $x + np \in$ [belong to] the domain of $f(x)$. Note that *p* is the *smallest* positive number for which this is true. You can also go backwards.

$$f(x) = f(x-p) = f(x-2p) = \ldots f(x-np)\ldots$$

where all the $x - np$ also belong to the domain. Usually, the domain is taken as all the real numbers $\mathbb{R}$.

## SINUSOIDAL PERIODIC FUNCTIONS

The most familiar examples of periodic functions are the **sinusoids**, such as

$$\sin x, \cos x, \sin(ax+b), a\sin x + b\cos x$$

and others.

**TIP:** Students should be asked to come up with some more examples of sinusoids, and find their periods. For example, $\cos(ax+b)$ or $a\sin x$.

Note: Wave theory is best suited to arguments in *radians*. We will assume this is the case hereafter, unless otherwise mentioned. For the simpler forms like $\sin x$ the period is

$$p = 2\pi$$

That is true for the linear combination

$a\sin x + b\cos x$ . [This can be shown by using the identity $\cos x = \sin(\pi/2 - x)$ .]

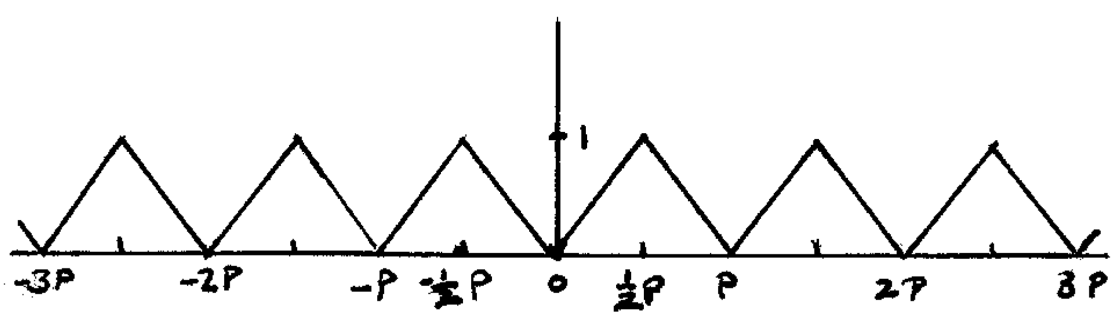
For a function like $\sin(ax+b)$

$$ap = 2\pi \to p = 2\pi/a$$

## NON-SINUSOIDAL PERIODIC FUNCTIONS

Let's dive into some examples.

Sawtooth Wave:




If you wish to be more "algebraic"

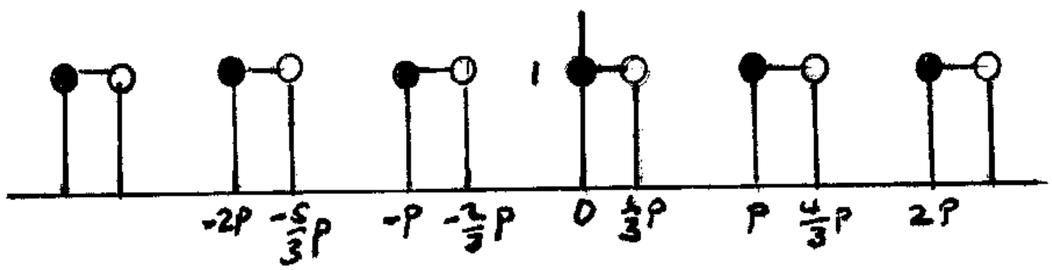
$$f(x) = \frac{2}{p}\left(x - np\right), np \le x < \left(n + \tfrac{1}{2}\right)p$$

$$f(x) = 2\left[1 - \left(\frac{x - np}{p}\right)\right], \left(n + \tfrac{1}{2}\right)p \le x < (n + 1)p$$

Here, *n* is any integer.

**TIP:** You may wish to have your students get these expressions. Formal mathematical induction is not needed. Derive for $n = 0$ and use the translations $x \to x - np$ for the other intervals.

Square Wave:




**TIP:** Have students supply the algebraic definition – this one is a little easier than for the sawtooth wave.

Here is the result.

$$f(x) = 1, np \le x < \left(n + \tfrac{1}{3}\right)p$$

$$f(x) = 0, \left(n + \tfrac{1}{3}\right)p \le x < (n + 1)p$$

**TIP:** Have students come up with other periodic functions.

## TRAVELING VS. STANDING WAVES

Before getting further into sinusoidal and other traveling waves, we can make a preliminary distinction between **traveling waves** and **standing waves**. Loosely, a *traveling wave* is a wave moving in only one direction. Still informally, a *standing wave* is one that is "standing still" – no surprise! In fact it is produced by the addition of two or more standing waves, moving in *different* directions.

**Note for sample: There is some material from the end of Appendix C on this page in the full document. Also there is some material from p. 46 at the bottom of this page.**

## APPENDIX D – ROGUE WAVES – AT SEA AND IN HARBORS

This appendix is an add on; it was not planned for originally. However, it occurred to me that, having heard only of rogue waves at sea, could they also occur in *enclosures* – such as harbors, marinas or bays? I thought that if there was some energy source, such as a river or an opening to ocean waves, an enclosed area might support the build up of large amplitude waves – enough to damage boats and docks, or even cause injuries.

I did some research and found that such effects can occur. Here, I will start by saying something about tsunamis and rogue waves that originate in out in the ocean. Then I will concentrate more on what happens or can happen in enclosures.

### Tsunamis

These are one of the most well known of damaging waves, especially since the devastation of December 2004 in Asia and to some extent in Africa. They are not tidal waves (although the name from Japanese means that and is a misnomer that has stuck). They are predominately caused by undersea seismic events, often involving a sudden rise of fall of the ocean floor. It is also conceivable that a large asteroid hitting the ocean could produce one. It is not strictly a "rogue wave" since its cause is well known and after its occurrence it's the timing of its spread can be predicted and warnings issued.

One of the key parameters is the speed of surface water waves. A general expression for its square is

$$v^2 = \frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi d}{\lambda}\right)$$

where *d* is the water's depth, $\lambda$ is the wavelength and *g* is the acceleration of gravity at sea level (about 9.81 m/s$^2$). For those unfamiliar with it, the hyperbolic tangent of *x* is

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

(Source: Antrim) The same source gives shallow and deep water limits

$$v^2 \approx gd, (d/\lambda < 0.05)$$

$$v^2 \approx g\lambda/2\pi, (d/\lambda > 0.5)$$

Note the independence of depth in the deep water limit and independence of wavelength for shallow water.