RIPPLE EFFECT, MULTIPLIER EFFECT ECONOMICS AND THE GEOMETRIC SERIES

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Level: Approximately 10th grade to college – courses in economics and mathematics

Purpose: To show the connection between the hot topic of the "ripple effect" in economics and the mathematics of the geometric series.

INTRODUCTION

Nowadays, with the economic crisis which emerged fully by 2008, we keep hearing about the **ripple effect** or sometimes the **multiplier effect**. Most people have at least an intuitive understanding of it – one action in an economy can cause a larger effect than the original action. The "ripple" means, for example, that if an autoworker gets a raise, he will spend more of his disposable income in a supermarket; the supermarket can afford to pay its employees more (or hire more of them); the supermarket employees spend more at the local mall; and so on.

The same concept can have a negative effect. If one kind of income drops, the overall effect on the economy is a still greater loss.

In my online research, I found little coverage of how the multiplier effect is related to the geometric series, other than mentioning it. A goal of this short lesson plan is to make the connection between the economic model and the math behind the estimated overall multiplier effect – filling in the gap. In other words, both learn some math and see it applied to the real world around swirling around us!

I will try to present this as a process of leading your students to *discovery* of the relationship between topics in economics and mathematics.

ECONOMICS BACKGROUND

The Keynesian Model

What I discuss comes from what is known as the **Keynesian Model** for **Aggregate Demand**. (References for print and online sources or listed later and may be consulted for more details.)

The model has two basic assumptions:

1. **Consumption** (i.e., spending) mainly comes from the part of the current take-home pay – what is available after necessities (such as food and rent) are paid – called **disposable income**.

2. If people get additional income, they do not spend it all – some is *saved*.

The second assumption is usually quantified by assuming that the additional income leads to additional spending or consumption that is a *constant fraction of the additional disposable income* – called the **marginal propensity to consume (*MPC*)**. This means that all people or businesses spend this same fraction. (We will look later at what happens if this assumption is not reasonable.) The remaining or unspent disposable income is **saved**. The fraction of the extra disposable income that is saved is called the **marginal propensity to save (*MPS*)**.

Some Mathematical Analysis of the Keynesian Model

For an analytical viewpoint, we can supply symbols for some of the terms.

DI = disposable income
C = consumption
S = saving

These are related by

$$DI = C + S$$

Note that we do not include the necessary expenditures in *C*. These are paid for by the part of the income that is not "disposable".

For any *change* in a quantity, we can put the Greek letter for capital delta – Δ – a common notation. A change in disposable income (positive or negative) is divided between changes in consumption and savings. Mathematically

$$\Delta DI = \Delta C + \Delta S$$

From their definitions we have

$$MPC = \frac{\Delta C}{\Delta DI}$$

$$MPS = \frac{\Delta S}{\Delta DI}$$

From these it is easily seen that

$$MPC + MPS = 1$$

That is, the two marginal propensities are complementary. We also see that "*MP*" is a ratio between two "deltas".

Example: Let's say somebody's annual disposable income was $30,000 and (because of a salary raise) it became $32,000. Then $\Delta DI = \$32{,}000 - \$30{,}000$.

The following table is based on assuming that $MPC = 0.6$. It is assumed also that a disposable income of $30,000 corresponds to a consumption of $35,000. This means that

S = *DI* - *C* = $30,000 - $35,000 = -$5,000. Negative savings is a case where one uses *debt* to consume/spend more than what is really available from current earnings. When this attitude becomes pervasive it can – and *has* – led to the current economic crisis! The table shows what happens under the above assumptions at various levels of disposable income.

Disposable Income *DI*	Consumption *C*	Saving *S*
$30,000	$35,000	-$5,000
$31,000	$35,000 + 0.6($1,000) = $35,600	-$5,000 + 0.4($1,000) = -$4,600
$32,000	$35,600 + 0.6($1,000) = $36,200	-$4,600 + 0.4 ($1,000) = -$4,200
...	...	...
$40,000	$35,000 + 0.6($10,000) =$41,000	-$5,000 + 0.4($10,000) = -$1,000
$50,000	$47,000	+$3,000
$60,000	$53,000	+$7,000

Note: These quantities can be graphed as straight line functions. See the suggested activities for more on this.

THE MULTIPLIER EFFECT

So far, I have discussed the effect of changing the disposable income of one individual household. The same ideas apply to an individual business.

Now consider the effect of additional spending by one consumer on other consumers – families or businesses. We can view this as a kind of "chain reaction" or **ripple effect**. As in the introductory examples, increased disposable income at any individual step results in increased spending of $\Delta C = MPC \cdot \Delta DI$. The increased spending at that step becomes the increased disposable income at another step. The next step will generally be the businesses that the family purchases from. (In a barter economy, it might be other families.)

The following treatment is my idea – not to say no one ever did it, just that I thought of it in developing this lesson plan.