PARABOLIC SKIS – ARE THEY REALLY PARABOLIC? HOW TO VERIFY OR DEBUNK MATHEMATICAL MODELS

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Level: This depends more on degree of preparation than grade level. It should generally be accessible from 11th grade through college.
Purpose: To learn how to test whether proposed mathematical models are applicable to reality

MOTIVATION

We often hear claims. As mathematical literates ("numerates"?) we should be able to question statements such as: The human population is growing exponentially, the length of the day is a sinusoidal function of time, or whether parabolic skis really are parabolic!

The goals of this lesson are:
- ☐ Learn to question statements about models
- ☐ Learn how to do your own measurements – to get data to test a verifiable claim
- ☐ Learn how to fit data by one or more models
- ☐ Analyze the results for goodness of fit and significance

Tips are given where appropriate.

DESIRED BACKGROUND

Like many of my products, you can take this at a variety of levels. Here are desirable basic backgrounds:
- Elements of curve fitting and regression
- A basic understanding of hypotheses
- The elements of goodness of fit and significance testing

The last is probably least important and can be learned "on the job".

MODEL

These are two equivalent ways to express a parabola.

$$y = f(x) = ax^2 + bx + c$$

$$y = f(x) = y_{min} + d(x - x_{min})^2$$

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The first form is the standard quadratic form. The second may be easier to visualize in the present application.

TIP: You may want to have your students show their equivalence in their own way at this point. One way is to start with the simpler form $y = kx^2$ and use the translations

$$x \rightarrow x - x_{min}$$

$$y \rightarrow y - y_{min}$$

which shifts the vertex from the origin to (x_{min}, y_{min}).

Equivalence requires

$$y = f(x) = y_{min} + kx^2 - 2kx_{min}x + kx_{min}^2$$

This yields

$$a = k$$

$$b = -2kx_{min}$$

$$c = y_{min} + kx_{min}^2$$

These can be inverted to get the parameters of the second form

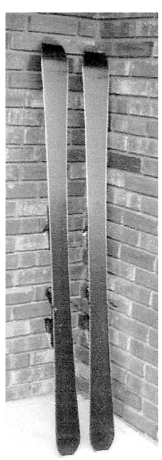
$$k = a$$

$$x_{min} = -\frac{b}{2a}$$

$$y_{min} = c - \frac{b^2}{4a}$$

MEASUREMENTS

I will outline what I did with a pair of skis. They were Rossignol 160 cm skis.



The photograph was taken viewed from the bottom.

I chose to start measuring length parallel to the tip to tail direction, starting at the widest part of the tip. (Further ahead than this, the tips start to turn up.) I put masking tape at 10 centimeter intervals, starting at the widest point, defined as $x = 0$ cm, and continuing with $x = 10, 20, 30. ...,$ 130 cm. I also placed another marker at the widest point of the tail, where it starts to turn up. This position was measured at $x = 137.55$ cm. All measurements were estimated to the nearest 0.01 cm (1/10 mm). Of course, I did not expect the last figure to more than a crude estimate. Actual errors were probably of the order of 0.05 cm.

I was concerned about possible cumulative error of the tape positions, as I did not have a long enough metric tape measure. I checked this by doing an overall measurement from the $x = 0$ marker to the one at 130 cm, using a metal tape measure marked in inches, resulting in 51 ½ inches. Multiplying this by 2.540 gave 130.81 cm. So, the marker placements were fairly accurate, and I kept them as is.

Next, came measuring the width of the ski at each marker. Half of these values gave the "y-values" for measurements from the longitudinal axis of symmetry. (The metal edges were included in the width measurements.)

The photo suggests that the curve is not symmetric about a position halfway between the tip and tail. Notice at the two widest points, the tip width appears greater.

Values of the parameters x_{min} & y_{min} can be found empirically, at least in principle. Of these, y_{min} is easy to get with confidence in its accuracy. However, x_{min} is hard to determine with great confidence, because the presumed parabola is so shallow.

MY MEASUREMENTS

The table gives the results at 15 points – 0 to 130 and the extra one at 137.55 cm. The y_i values are the results of dividing the width measurements by two.

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