INTRODUCTION TO PERCOLATION THEORY – WITH APPLICATION TO FOREST FIRE SPREAD AND PREVENTION– A LESSON PLAN

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Level: This lesson plan can be used in a wide range of levels. It can be part of a math course, but also fits into courses like physics, earth science, etc. Minimal math background is needed – mostly an elementary grasp of probability.
Purpose: To get students used to thinking of how models can be used for real world applications and to be able to criticize the appropriateness of proposed models to the particular situation.

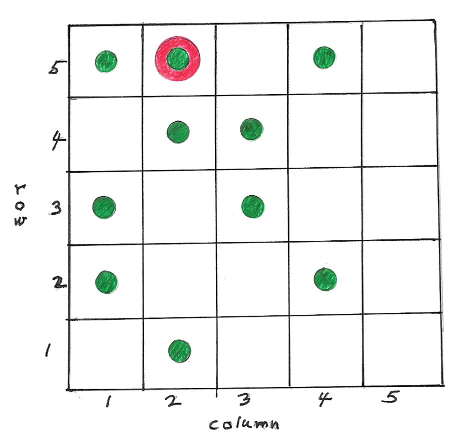
INTRODUCTION

In a recent article in *Scientific American* (see References, SciAm), the problem of the spreading of forest fires is discussed in detail. One of the ways experts on the subject deal with it is by the development and use of **models** of how these fires spread. Such models can lead to **simulations** on a computer rather than by doing experimental burns. One such approach is **percolation theory** or *stochastic percolation*. It is not considered the best way to model forest fires these days. Nevertheless, it is easy to understand in its most basic form. Also, it has applications to many other phenomena, including: the spread of epidemics, growth of large molecules, distribution of oil reserves, and electrical conductivity. (See Reference DS&AA).

In this lesson plan, students will do their own simulations of the spread of forest fires using percolation theory. The only tools needed are: any random number generator (most scientific or graphing calculators) or tables, graph paper, and colored pens or pencils (red, green and black). They will be encouraged to think of creative applications of such models and even to come up with other ways to use them.

SOME BASIC IDEAS OF PERCOLATION THEORY AS APPLIED TO FOREST FIRES

Consider the figure shown here.

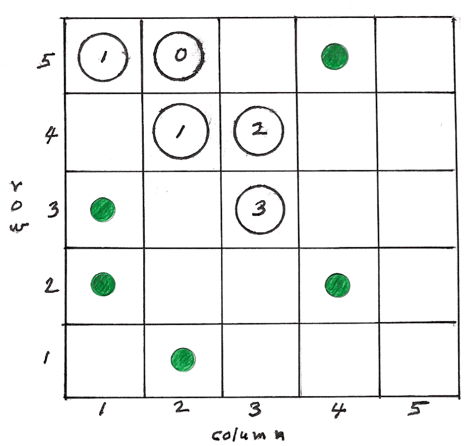


It shows a five by five grid. Ten of the 25 squares have green dots. These represent the original locations of *living trees*. One of these is shown surrounded by a red circle. Its location is at the (row,column) coordinates (5,2). This represents a tree that has just been ignited, such as by a lightning strike.

To continue the model, we must make assumptions on how the fire spreads from an ignited tree. We will start with the simplest one: The fire spreads only to trees that are so called "**nearest neighbors"** of an ignited tree. These are trees that are in squares *adjacent* to the ignited tree's square – in a horizontal or vertical direction.

We will also assume that just after a fire spreads to a neighboring tree, the originally ignited tree burns up. This will be shown by a black circle around the red one. The process is made easiest to follow by imagining that we have a "clock" that ticks once whenever a tree is ignited and next time when that tree is burned-out. This allows us to keep track of the fire. Putting a number inside the red circles that correspond to the time at which it was ignited gives us a complete *history* of the progress of the fire.

The next figure show the history of the fire that was started in the tree at (5,2).



The diagram gives useful information:

- The fire lasted for three time units before burning out.
- Five of the 10 trees were consumed.

USING RANDOM NUMBERS TO SIMULATE FOREST FIRES

How can you arrive at the initial tree configuration in the example above? One of the *key concepts* in percolation theory is the *probability that a site is occupied*. Often the initial state of the system (which sites are occupied at first) is assumed to be *random*. (Later we will look at some other possibilities.) You can simulate the initial configuration for a given occupation probability *p* by simply generating $L \times L$ random numbers in the usual range of (0,1). Such numbers are readily available on most scientific and graphing calculators, spread sheets, and other mathematical or statistical software. Each of the random numbers is assigned to one of the sites as denoted by (row,column) coordinates in some systematic way like $(1,1),(1,2),\ldots(1,5),(2,1),(2,2)\ldots(5,4),(5,5)$ for $L = 5$, for the 25 numbers generated. (On the TI-83 Plus graphing calculator, MATH, PRB, 1:rand(25) does the job.) For each value that is less than the value *p*, a tree (green spot) is placed in the site. Otherwise it is left blank. (Have your students become convinced that there is no practical need to say that the value must be less than *or equal* to *p*.)

To simulate the ignition or start of the fire, we will consider the case of a random strike by lightning. For this you can generate several *random integers* in the range of $1,2,\ldots L^2$. For $L = 5$, this is 25 values. Each integer corresponds to a (row,column) position by a scheme such as is shown in the table.