CENTER OF GRAVITY AND TORQUE – WITH APPLICATIONS TO STABILITY AND TIPPING OF OBJECTS – A LESSON PLAN

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Level: Any physics (or math) class for students who have learned the elements of mechanics and have at least an intuitive idea of the torque concept.

Purpose: To get students to think of how physics can be applied to designing everyday objects.

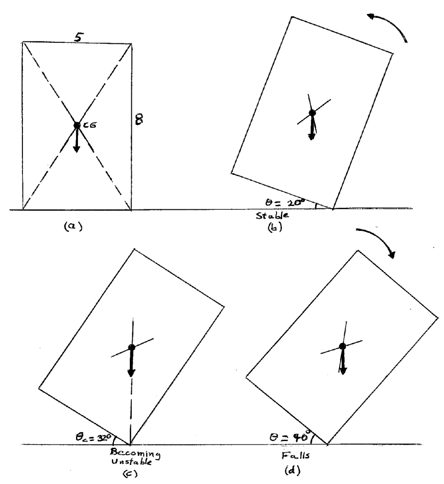
INTRODUCTION

Ask your students to imagine that they live in a seismically active area (a fancy way of saying that earthquakes are likely). Various items are on shelves or a mantel. They should think of actual objects they have at home or see in school and think how likely they are to *tip over* in the event that they are displaced from their normal positions. They can also try it experimentally, where it is safe to do so. We will assume that each object has a flat base and it rests on a flat horizontal surface.

Although, in general this can be a complicated problem – depending on how the object is displaced, one case is easy to solve: Assume the object is displaced by rotation about a horizontal axis within it. Also, assume that one side (or at least a point) of the base remains in contact with the surface on which it rests and the surface itself does not move or vibrate. Finally, it must be possible to locate the *center of gravity*. (Symmetry often helps with the last.) The problem we want to solve is to see how far the object can be slowly tipped until it becomes *unstable* and can *fall over*. Obviously, not all these simplifications apply to a situation like an earthquake or storm. This should be made clear to the students before going on to the simpler case considered first.

BASIC CONCEPTS

The following figure shows an object with a rectangular cross section as viewed from one side. It could be a rectangular solid or a cylinder for example.



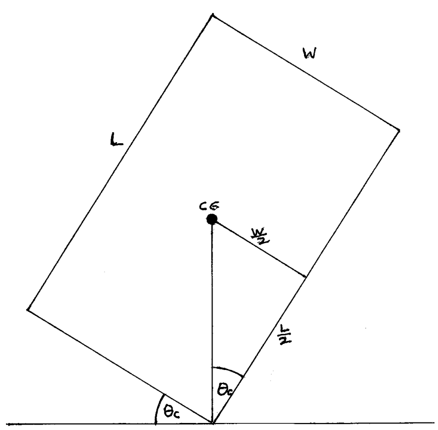
(a) shows the object at rest. (b) shows it tipped by a small amount. (c) shows it tipped to where its center of gravity is over the edge of the base on the supporting surface. (d) shows it tipped past that point. The straight arrows suggest forces. The curved arrows indicate expected rotations when the object is released in each position. The angles marked θ are the amount of tipping. In (c) it is marked θ_c to show a *critical angle*. Construction lines that show locating the center of gravity at the intersection of the two diagonals are left in.

The **center of gravity** or **center of mass** is shown. These concept will be introduced in the lesson plan. An option would be to do that now. But, I will assume they have an intuitive idea of their meaning.

The **torque** concept – especially with its *sense* can be used for inferring the direction of the resulting angular rotation. (See Appendix A for additional background.)

SIMPLE EXAMPLE – RECTANGULAR CROSS SECTION, UNIFORM DENSITY

First, the concept of mass density should be known or taught. Students should be encouraged to realize that the critical angle is related to the *ratio* of dimensions, *not* on their absolute values. Also, they should realize that symmetry allows putting the center of gravity (CG) at the center of the figure. This position can be located either by its position halfway from either side ($L/2$ and $W/2$) in the figure below, or by the intersection of the diagonals of the rectangle – as done in the first figure. (Here is an opportunity to refer to the properties of rectangles, including that their diagonals bisect, to show the two ways are equivalent. See Appendix B for details.)



The key point is that the critical angle occurs when the CG is directly over the point of contact of the edge of the base and the supporting surface. Try to get them to relate this to zero torque and where the torque starts to change direction.   Also, try to get the students to come up with something like the diagram. The result of analyzing it is

$$\tan\theta_c = \frac{W/2}{L/2} = \frac{W}{L}$$

giving

$$\theta_c = \tan^{-1}\left(\frac{W}{L}\right)$$

This should help them to see that:

- As expected, the critical angle only depends on the *ratio* of the dimensions.
- Stability is increased by making the base width to object height ratio larger – giving a larger critical angle.

At this point they have learned some elements of design.

SOME POSSIBLE EXPERIMENTS TO DO

☐ Locate or cut blocks of wood in the shape of rectangular solids (parallelopipeds). Use solid wood, such as pine, but without knotholes, rather than plywood, to get the density as uniform as possible.

☐ Measure all three dimensions (*L,W,T)*. Use centimeters or millimeters, as it produces decimal remainders that are easier to use in calculations. Measure at more than one location to check for consistency.

☐ Find, or construct, a protractor whose center is on a straight edge. This allows aligning the center with the “pivot” at the bottom of each block.

☐ Work with at least two people: one two locate the “tipping point” or critical rotation, the other to measure the critical angle experimentally. An option is digital photos.   Note: Do *not* do the theoretical calculation of θ_c first. You want to avoid biasing the observations.

☐ Try the observations using four different corners to check for consistency.

☐ Now do the theoretical calculations for the critical angle and compare to the *empirical* findings. (Here is a good time to introduce that word if not known.)

☐ If the first four cases were done with *L* and *W* as the dimensions involved, repeat with *L* and *T* and with *W* and *T*.